

GATE 2009 SOLUTIONS

AEROSPACE ENGINEERING

IIT Roorkee
Organizing Institute

Solution By Team IGC
Special Thanks to Mr Bhajan Lal J and Mr Sasmit Sanjay

Q. 1 – Q. 20 carry one mark each.

Q.No. 1 For a flow through a Prandtl-Meyer expansion wave

- (A) Mach number stays constant. (B) Entropy stays constant.
(C) Temperature stays constant. (D) Density stays constant.

Sol. B

Prandtl – Meyer expansion wave is an isentropic process. So entropy is constant.

Mach number increases and temperature, pressure and density decreases across wave.

Q.No. 2 For two-dimensional irrotational and incompressible flows

- (A) Both potential and stream functions satisfy the Laplace equation.
(B) Potential function must satisfy the Laplace equation but the stream function need not.
(C) Stream function must satisfy the Laplace equation but the potential function need not.
(D) Neither the stream function nor the potential function need to satisfy the Laplace equation.

Sol. A

Note – Go through potential and stream function properties.

Q.No. 3 A trailing edge plain flap deflected downward increases the lift coefficient of an airfoil by

- (A) Increasing the effective camber of the airfoil.
(B) Delaying the separation of the flow from the airfoil surface.
(C) Increasing the local airspeed near the trailing edge.
(D) Controlling the growth of the boundary layer thickness along the airfoil surface.

Sol. A

A plain trailing edge flap mainly increases the effective camber of the airfoil when deflected downwards.

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Due to decreasing local radius of curvature, local centrifugal force increases and in result of local pressure increase upward in normal direction down to the airfoil.

Note – Go through all trailing edge flaps and their properties.

Q.No. 4 Thin airfoil theory predicts that the lift slope is $\frac{dc_l}{d\alpha} = 2\pi$ for

- (A) Symmetric airfoils only.
(C) Any airfoil shape.

- (B) Cambered airfoils only.
(D) Joukowski airfoils only.

Sol. C

Q.No. 5 The ordinary differential equation $\frac{d^2y}{dx^2} + ky = 0$ where k is real and positive

- (A) is non-linear
(B) has a characteristic equation with one real and one complex root
(C) has a characteristic equation with two real roots
(D) has a complementary function that is simple harmonic

Sol. D

$\frac{d^2y}{dx^2} + ky = 0$ where k is real and positive.

$$(D^2 + k)y = 0$$

$$m^2 + k = 0$$

$$m = +i\sqrt{k}, -i\sqrt{k} \rightarrow \text{two complex roots}$$

Complementary function -

$$y = C_1 \cos(\sqrt{k}x) + C_2 \sin(\sqrt{k}x)$$

So it is simple harmonic function.

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Q.No. 6 A non-trivial solution to the $(n \times n)$ system of equations $[A]\{x\} = \{0\}$, where $\{0\}$ is the null vector

- (A) can never be found
- (B) may be found only if $[A]$ is not singular
- (C) may be found only if $[A]$ is an orthogonal matrix
- (D) may be found only if $[A]$ has at least one eigenvalue equal to zero

Sol. D

For non – trival solution,

$$|A| = 0$$

$$\rightarrow \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \dots \dots \lambda_n = 0$$

Hence at least one eigenvalue has to be equal to zero.

Q.No. 7 For a plane strain problem, the stresses satisfy the condition

- (A) $\tau_{xz} = \tau_{yz} = \sigma_z = 0$
- (B) $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \nu(\sigma_x + \sigma_y)$
- (C) $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \nu\tau_{xy}$
- (D) $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \nu(\sigma_x + \sigma_y) + (1-\nu)\tau_{xy}$

Sol. B

For a plane strain in x-y plane,

$$\epsilon_{xx}, \epsilon_{yy}, \gamma_{xy} \neq 0$$

$$\epsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$$

$$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy} \neq 0$$

$$\tau_{xz} = \tau_{yz} = 0$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \nu\left(\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E}\right) = 0$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$$

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Q.No. 8 The propulsive efficiency of a turbo-jet engine moving at velocity U_∞ and having exhaust velocity U_e with respect to the engine is given by

- (A) $\frac{2}{U_\infty/U_e + 1}$ (B) $1 - \frac{U_\infty}{U_e}$ (C) $\frac{2U_\infty U_e}{U_e^2 + U_\infty^2}$ (D) $\frac{2U_\infty}{U_e + U_\infty}$

Sol. DHint: Propulsive efficiency of turbojet engine, $\eta_p = \frac{\text{thrust power}}{\Delta(K.E.)}$

Q.9 An aircraft is flying at $M = 2$ where the ambient temperature around the aircraft is 250 K. If the specific heat ratio for air $\gamma = 1.4$, the stagnation temperature on the surface of the aircraft is

- (A) 200 K (B) 450 K (C) 350 K (D) 1450 K

Sol. B

$$M = 2, T = 250 \text{ K}, \gamma = 1.4$$

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

$$T_0 = 450 \text{ K}$$

Q.No. 10 The division of feed air to an aircraft gas-turbine combustor into primary and secondary streams serves which of the following purposes ?

- P. a flammable mixture can be formed
Q. cooling of combustor liner and flame tube can be accomplished
R. specific fuel consumption can be reduced

- (A) P and R (B) Q and R (C) P and Q (D) P, Q and R

Sol. C

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Q.No. 11 Classify the following propellants as: cryogenic (C) , semi-cryogenic (SC), compressed gas (CG) and earth storable (ES)

N_2O_4 -UDMH (nitrogen tetra oxide and unsymmetrical di-methyl hydrazine)

LOX-RP1 (liquid oxygen and kerosene)

LOX-LH₂ (liquid oxygen and liquid hydrogen)

N_2 (nitrogen gas)

- (A) N_2O_4 -UDMH (ES), LOX-RP1 (C), LOX-LH₂ (C), N_2 (C)
(B) N_2O_4 -UDMH (SC), LOX-RP1 (SC), LOX-LH₂ (C), N_2 (C)
(C) N_2O_4 -UDMH (ES), LOX-RP1 (SC), LOX-LH₂ (C), N_2 (CG)
(D) N_2O_4 -UDMH (ES), LOX-RP1 (C), LOX-LH₂ (C), N_2 (CG)

Sol. C

Q.No. 12 A conventional altimeter is a

- (A) Pressure transducer (B) Temperature transducer
(C) Density transducer (D) Velocity transducer

Sol. A

Altimeter measures altitude from mean sea level by using static pressure.

Q.No. 13 The relation between an airplane's true airspeed V_{TAS} and equivalent airspeed V_{EAS} in terms of the density ratio ($\sigma = \frac{\rho}{\rho_0}$), where ρ_0 is the air density at sea-level and ρ is the air density at the altitude at which the airplane is flying, is given by the formula:

- (A) $\frac{V_{EAS}}{V_{TAS}} = \sigma$ (B) $\frac{V_{EAS}}{V_{TAS}} = \sigma^2$
(C) $\frac{V_{EAS}}{V_{TAS}} = \sqrt{\sigma}$ (D) $\frac{V_{EAS}}{V_{TAS}} = \frac{1}{\sqrt{\sigma}}$

Sol. C

We know,

$$\frac{1}{2}\rho V_{TAS}^2 = \frac{1}{2}\rho_0 V_{EAS}^2$$

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$$\frac{V_{EAS}}{V_{TAS}} = \sqrt{\frac{\rho}{\rho_0}} = \sqrt{\sigma}$$

Q.No. 14 An unswept fixed-winged aircraft has a large roll stability if the wing is placed

- (A) low on the fuselage and has negative dihedral angle
- (B) low on the fuselage and has positive dihedral angle
- (C) high on the fuselage and has negative dihedral angle
- (D) high on the fuselage and has positive dihedral angle

Sol. B

Low wing a/c with positive dihedral angle gives more restoring roll moment and stabilizes spiral mode.

Q.No. 15 Thrust available from a turbojet engine

- (A) increases as altitude increases
- (B) increases up to the tropopause and then decreases
- (C) remains constant at all altitudes
- (D) decreases as altitude increases

Sol. D

$$\text{Thrust, } F = \dot{m}(V_e - V_a) + A_e(P_e - P_a)$$

$$\text{So, } F \propto \dot{m}$$

$$F \propto \rho \quad (\text{Mass flow rate, } \dot{m} = \rho AV)$$

Now density decrease with altitude. So Thrust will also decrease with altitude.

Q.No. 16 If $C_{m_{CG}}$ is the pitching moment coefficient about the center of gravity of an aircraft, and α is the angle

of attack, then $\frac{dC_{m_{CG}}}{d\alpha}$ is

- (A) a stability derivative which represents stiffness in pitch
- (B) a stability derivative which represents damping in pitch
- (C) a control derivative in pitch
- (D) positive for an aircraft that is stable in pitch

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Sol. A

$$\frac{dC_{mCG}}{d\alpha} < 0 \rightarrow \text{For static longitudinal stability}$$

$$C_m = C_{m_\alpha} \cdot \alpha$$

So, C_{m_α} represent stiffness in pitch

Q.No. 17 The life of a geo-stationary communications satellite is limited by

- (A) the working life of the on-board electronic circuitry
- (B) the time it takes for its orbit to decay due to atmospheric drag
- (C) the quantity of on-board fuel available for station-keeping
- (D) the number of meteorite impacts that the satellite structure can withstand before breaking up

Sol. C

Q.No. 18 For a critically damped single degree of freedom spring – mass – damper system with a damping constant c of 4 Ns/m and spring constant k of 16 N/m, the system mass m is

- (A) 0.5 kg
- (B) 0.25 kg
- (C) 2 kg
- (D) 4 kg

Sol. B

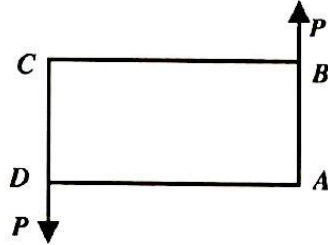
$$C = 4 \text{ Ns/m}, k = 16 \text{ N/m}$$

$$C = C_c = 2\sqrt{km} \quad (\text{For a critical damped system})$$

$$m = 0.25 \text{ kg}$$

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Q.No. 19 In a thin walled rectangular tube subjected to equal and opposite forces P as shown in the figure, the shear stress along leg AB is



- (A) zero
(B) constant non-zero
(C) varies linearly
(D) varies parabolically

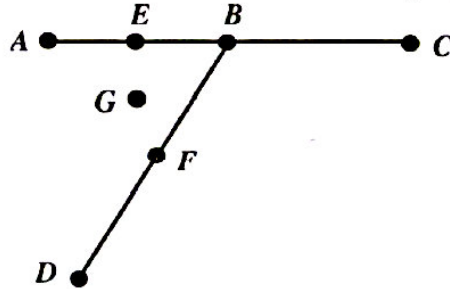
Sol. B

Here couple create torque and given section undergo twist, for thin-walled sections

$$T = 2Aq = 2A\tau t$$

$$\tau = \frac{T}{2At} \rightarrow \text{Constant}$$

Q.No. 20 For the thin walled beam cross section as shown in the figure, the shear centre lies at



- (A) Mid point of AB , i.e. at point E
(B) Mid point of BD , i.e. at point F
(C) Junction point B
(D) at a point G lying within the area ABC

Sol. C, shear flow for a section with common junction is always junction point

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Q. 21 to Q. 60 carry two marks each.

Q.No. 21 Let M_0 be the total mass of a single stage rocket, M_p be the total mass of propellant, M_L be the mass of payload carried by the rocket and M_S be the mass of inert structural components. If I_{sp} is the specific impulse of the propulsion system (in seconds) and g is the acceleration due to gravity, then the maximum velocity that can be attained by the rocket vehicle in the absence of gravity and atmospheric drag is given by

(A) $gI_{sp} \ln\left(\frac{M_0}{M_p}\right)$

(B) $gI_{sp} \ln\left(\frac{M_0}{M_L + M_S} - 1\right)$

(C) $gI_{sp} \ln\left(\frac{M_0}{M_S}\right)$

(D) $gI_{sp} \ln\left(\frac{M_0}{M_0 - M_p}\right)$

Sol. D

For a Rocket vehicle -

From Newton's 2nd law of motion,

$$F = ma$$

$$m \cdot \frac{dv}{dt} = F - D - mg \sin \theta \dots (1)$$

Here, F = thrust provided by rocket engine in velocity direction

D = Drag in opposite direction to velocity of rocket

θ = Angle made by rocket velocity to the horizontal direction

Thrust, $F = \dot{m}c$

Here, \dot{m} = mass flow rate = $-\frac{dm}{dt}$

$C = V_e$ = Gas velocity

From equation (1) in the absence of gravity and atmospheric drag,

$$m \cdot \frac{dv}{dt} = F = -\frac{dm}{dt} \times C$$

$$dV = -\frac{dm}{m} \cdot C$$

$$\int_{V_1}^{V_2} dV = -C \int_{M_i}^{M_f} \frac{dm}{m}$$

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$$V_2 - V_1 = C \cdot \ln \frac{M_i}{M_f} \dots\dots (2)$$

$$\text{Specific impulse, } I_{SP} = \frac{F}{\dot{m}g} = \frac{\dot{m}C}{\dot{m}g} = \frac{C}{g}$$

From equation (2),

$$\Delta V = I_{SP} \cdot g \cdot \ln \frac{M_i}{M_f}$$

$$(\Delta V)_{max} = I_{SP} \cdot g \cdot \ln \left(\frac{M_0}{M_0 - M_p} \right)$$

Q.No. 22 An ideal axial compressor is driven by an ideal turbine across which the total temperature ratio is 0.667. If the total temperature at turbine inlet is $T_0 = 1500$ K and specific heat of gas $c_p = 1$ kJ/kg/K, the power drawn by the compressor per unit mass flow rate of air is approximately

- (A) 300 kW/kg/s (B) 1000 kW/kg/s (C) 600 kW/kg/s (D) 500 kW/kg/s

Sol. D

For ideal turbine,

$$T_{03} = 1500 \text{ K, } C_p = 1 \text{ kJ/kg-K}$$

$$\frac{T_{04}}{T_{03}} = 0.667$$

$$\dot{W}_C = \dot{W}_T = \dot{m}_g \cdot C_p (T_{03} - T_{04})$$

$$\frac{\dot{W}_T}{\dot{m}_g} = C_p T_{03} \left(1 - \frac{T_{04}}{T_{03}} \right) = 1 \times 1500 \times (1 - 0.667) = 499.5 \text{ kW/kg/s}$$

Q.No. 23 The performance of a solid rocket motor is improved by replacing the old propellant with a new one. The new propellant gives a combustion temperature 40% higher than the previous propellant without appreciable change in molecular weight of combustion products and other operating parameters. By approximately what percentage is the specific impulse of the new motor higher than the old one ?

- (A) 18% (B) 96% (C) 42% (D) 112%

Sol. A

$$\text{Specific impulse, } I_{SP} = \frac{F}{\dot{m}g} = \frac{\dot{m}C}{\dot{m}g} = \frac{C}{g}$$

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Let's assume convergent nozzle used in rocket motor.

T_c is temperature in chamber.

$$h_0 = h + \frac{v^2}{2}$$

$$T_c = T + \frac{v^2}{2C_p}$$

$$V_e = \sqrt{2C_p(T_c - T)} = \sqrt{2C_p T_c \left(1 - \frac{T}{T_c}\right)}$$

$$\frac{T_c}{T} = 1 + \frac{\gamma-1}{2} M^2 = \frac{\gamma+1}{2} \quad (M=1)$$

$$V_e = C = \sqrt{2 \frac{\gamma R}{\gamma-1} T_c \left(1 - \frac{2}{\gamma+1}\right)} = \sqrt{2 \frac{\gamma R}{\gamma+1} T_c}$$

$$\text{Specific impulse, } I_{SP} = \frac{C}{g} = \frac{\sqrt{2 \frac{\gamma R}{\gamma+1} T_c}}{g}$$

$$I_{SP} \propto \sqrt{T_c}$$

$$I_{SP} = K\sqrt{T_c}$$

$$I_{SP,1} = K\sqrt{T_c + 0.4T_c} = 1.18 K\sqrt{T_c}$$

$$\% \text{ change in } I_{SP} = \frac{I_{SP,1} - I_{SP}}{I_{SP}} \times 100\% = 18\%$$

Q.No. 24 A solid rocket motor has an end burning grain of cross-sectional area $A_{CS} = 0.4 \text{ m}^2$. The density of propellant is $\rho_p = 1500 \text{ kg/m}^3$ and has linear regression rate $\dot{r} = 5 \text{ mm/s}$. If the specific impulse of the propulsion system is $I_{sp} = 200$ seconds, the thrust produced by the motor is approximately

- (A) 3 kN (B) 6 kN (C) 1.5 kN (D) 12 kN

Sol. B

$$A_{CS} = 0.4 \text{ m}^2, \rho_p = 1500 \text{ kg/m}^3, \dot{r} = 5 \text{ mm/s} = 5 \times 10^{-3} \text{ m/s}, I_{SP} = 200 \text{ seconds}$$

$$\dot{m}_p = \rho_p A_{CS} \dot{r} = 3 \text{ kg/s}$$

$$F = \dot{m}_p \cdot C = \dot{m}_p \cdot I_{SP} \cdot g = 3 \times 200 \times 9.81 = 5.88 \text{ kN} \cong 6 \text{ kN}$$

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Q.No. 25 An ideal ramjet is flying at an altitude of 10 km with a velocity of 1 km/s. The ambient pressure is 0.25 bar and temperature is 225 K. The exhaust gases from the engine are optimally expanded and leave the nozzle at 900 K. If the specific heat ratio (γ) remains constant, the specific thrust developed by the engine is approximately

- (A) 1000 N-s/kg (B) 2000 N-s/kg (C) 500 N-s/kg (D) 4000 N-s/kg

Sol. A

$$V_a = 1 \text{ km/s} = 1000 \text{ m/s}, T_a = 225 \text{ K}, P_a = 0.25 \text{ bar}, T_e = 900 \text{ K}, \gamma = 1.4$$

We know for an ideal ramjet engine,

$$M_a = M_e$$

$$\frac{V_a}{\sqrt{\gamma R T_a}} = \frac{V_e}{\sqrt{\gamma R T_e}}$$

$$V_e = V_a \sqrt{\frac{T_e}{T_a}}$$

$$\text{Thrust, } F = \dot{m} V_e - \dot{m} V_a$$

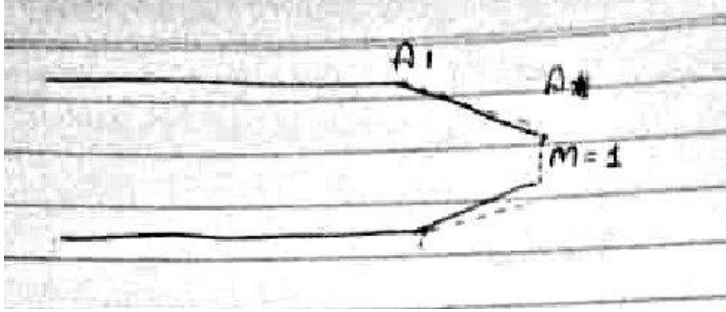
$$\frac{F}{\dot{m}} = V_e - V_a = V_a \sqrt{\frac{T_e}{T_a}} - V_a = V_a \left(\sqrt{\frac{T_e}{T_a}} - 1 \right) = 1000 \text{ N-s/kg}$$

Q.No. 26 A combat aircraft engine is equipped with an afterburner followed by a variable area convergent nozzle (operating with the nozzle choked). The exhaust gas temperature is 750 K when afterburner is off and 3000 K when it is on. When the afterburner is turned on, (assuming the total pressure remains the same, the mass of fuel added in the afterburner is negligible *i.e.*, the mass flow rate remains the same, and the specific heat ratio (γ) remains constant), approximately by what factor must the nozzle area be changed?

- (A) 0.5 (B) 4 (C) 1 (D) 2

Sol. D

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$P_0 = \text{const. throughout}$

$\dot{m}_{\text{before}} = \dot{m}_{\text{after}}$

$\gamma = \text{const.}$

$A_* = \text{variable, } A_1 = \text{const.}$

$T_{e1} = 750 \text{ K, } T_{e2} = 3000 \text{ K}$

Assume isentropic flow,

$\dot{m} = \rho_1 A_1 V_1 = \rho_{e1} A_{*1} V_{e1} = \rho_{e2} A_{*2} V_{e2}$

$V_e = \sqrt{\gamma R T_e}$ (choked)

$\rho_{e1} A_{*1} V_{e1} = \rho_{e2} A_{*2} V_{e2}$

$\frac{\rho_{e1}}{R T_{e1}} A_{*1} V_{e1} = \frac{\rho_{e2}}{R T_{e2}} A_{*2} V_{e2}$

$\frac{A_{*2}}{A_{*1}} = \frac{V_{e1} \rho_{e1} T_{e2}}{V_{e2} \rho_{e2} T_{e1}} = \frac{\sqrt{\gamma R T_{e1}} \rho_{e1} T_{e2}}{\sqrt{\gamma R T_{e2}} \rho_{e2} T_{e1}} = \sqrt{\frac{T_{e2} \rho_{e1} P_0}{T_{e1} \rho_{e2} P_0}} = \left(\frac{P_0}{P_{e2}}\right) \times \left(\frac{P_{e1}}{P_0}\right) \sqrt{\frac{T_{e2}}{T_{e1}}}$

$P_e \rightarrow$ Choked condition

$\frac{P_0}{P_e} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} \quad (M = 1)$

$\therefore \frac{P_0}{P_{e1}} = \frac{P_0}{P_{e2}} \quad (P_0 = \text{const. throughout})$

$\frac{A_{*2}}{A_{*1}} = \sqrt{\frac{T_{e2}}{T_{e1}}} = \sqrt{\frac{3000}{750}} = 2$

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Q.No. 27 An airplane flying at 100 m/s is pitching at the rate of 0.2 deg/s. Due to this pitching, the horizontal tail surface located 4 metres behind the centre-of-mass of the airplane will experience a change in angle of attack, which is

- (A) 0.01 deg (B) 0.008 deg (C) 0.04 deg (D) 0.004 deg

Sol. B

$$V_{\infty} = 100 \text{ m/s}$$

$$q = 0.2 \text{ deg/s}$$

$$l_t = 4 \text{ m}$$

Tangential velocity induced due to pitch up, $\omega = ql_t = 0.2 \times \frac{\pi}{180} \times 4 = 0.013962 \text{ m/s}$

$$\Delta\alpha = \tan^{-1}\left(\frac{\omega}{V_{\infty}}\right) = 0.008 \text{ deg.}$$

Q.No. 28 The contribution of the horizontal tail to the pitching moment coefficient about the center of gravity ($C_{m_{CG}}$) of an aircraft is given by $C_{m_{tail}} = 0.2 - 0.0215 \alpha$, where α is the angle of attack of the aircraft. The contribution of the tail to the aircraft longitudinal stability

- (A) is stabilizing
(B) is destabilizing
(C) is nil
(D) cannot be determined from the given information

Sol. A

$$C_{m_{tail}} = C_{m_0} + C_{m_{\alpha}} \cdot \alpha$$

$$C_{m_{tail}} = 0.2 - 0.0215 \alpha$$

$$\frac{\partial C_m}{\partial \alpha} = -0.0215$$

$$\frac{\partial C_m}{\partial \alpha} < 0 \rightarrow \text{a/c is longitudinal stable.}$$

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Q.No. 29 The linearized dynamics of an aircraft (which has no large rotating components) in straight and level flight is governed by the equations

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} \vec{x}$$

where $\vec{x} = [u \ w \ q \ \theta \ v \ p \ r \ \phi]^T$, $[]^T$ represents the transpose of a matrix, $[A]$, $[B]$, $[C]$ and $[D]$ are 4×4 matrices and $[0]$ is the 4×4 null matrix. Which of the following is true ?

- (A) $[A] \neq [0]$; $[B] \neq [0]$; $[C] = [0]$; $[D] \neq [0]$
- (B) $[A] = [0]$; $[B] \neq [0]$; $[C] \neq [0]$; $[D] = [0]$
- (C) $[A] \neq [0]$; $[B] = [0]$; $[C] = [0]$; $[D] \neq [0]$
- (D) $[A] \neq [0]$; $[B] = [0]$; $[C] \neq [0]$; $[D] = [0]$

Sol. C

For straight and level flight,

$w = 0$ m/s, $q = 0$ rad/s, $\theta = 0$ deg,

Q.No. 30 The velocity vector of an aircraft along its body-fixed axis is given by $\vec{V} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$. If V is the magnitude

of \vec{V} , α is the angle of attack and β is the angle of sideslip, which of the following set of relations is correct ?

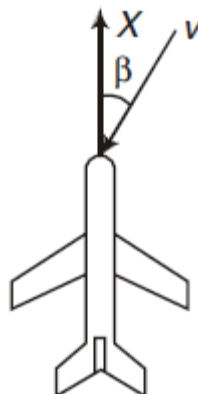
- (A) $u = V \sin \beta \cos \alpha$; $v = V \sin \beta$; $w = V \cos \beta \sin \alpha$
- (B) $u = V \cos \beta \cos \alpha$; $v = V \cos \beta$; $w = V \cos \beta \sin \alpha$
- (C) $u = V \cos \beta \cos \alpha$; $v = V \sin \beta$; $w = V \sin \beta \sin \alpha$
- (D) $u = V \cos \beta \cos \alpha$; $v = V \sin \beta$; $w = V \cos \beta \sin \alpha$

Sol. D

$$\vec{V} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

α = angle of attack

β = angle of sideslip



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$$v = V \sin \beta$$

$$u = V \cos \beta \cos \alpha$$

$$w = V \cos \beta \sin \alpha$$

Note – Go through equation of motion in dynamic stability.

Q.No. 31 An aircraft of mass 2500 kg in straight and level flight at a constant speed of 100 m/s has available excess power of 1.0×10^6 W. The steady rate of climb it can attain at that speed is

- (A) 100 m/s (B) 60 m/s (C) 40 m/s (D) 20 m/s

Sol. C

$$m = 2500 \text{ kg}$$

$$V = 100 \text{ m/s}$$

$$\text{Available excess power} = 1.0 \times 10^6 \text{ W}$$

$$R/C = V \sin \gamma = \frac{\text{excess power}}{W} = 40.77 \text{ m/s}$$

Q.No. 32 The acceleration due to gravity on the surface of Mars is 0.385 times that on earth, and the diameter of Mars is 0.532 times that of earth. The ratio of the escape velocity from the surface of Mars to the escape velocity from the surface of earth is approximately

- (A) 0.453 (B) 0.205 (C) 0.851 (D) 0.724

Sol. A

$$V_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$\rightarrow \frac{(V_{esc})_{Mars}}{(V_{esc})_{Earth}} = \frac{\sqrt{2g_m R_m}}{\sqrt{2g_e R_e}} = \sqrt{\frac{g_m R_m}{g_e R_e}} = \sqrt{0.385 \times 0.532} = 0.452$$

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Q.No. 33 Which of the following statements are true for flow across a stationary normal shock ?

- P. Stagnation temperature stays constant.
- Q. Stagnation pressure decreases.
- R. Entropy increases.
- S. Stagnation pressure increases.
- T. Stagnation temperature increases.

- (A) P, Q, R (B) Q, R, S (C) R, S, T (D) S, T, P

Sol. A

Note – Go through normal shock wave's properties.

Q.No. 34 A model airfoil in a wind tunnel that is operating at 50 m/s develops a minimum pressure co-efficient of -6.29 at some point on its upper surface. The local airspeed at that point is

- (A) 50 m/s (B) 125 m/s (C) 135 m/s (D) 150 m/s

Sol. C

$$V_{\infty} = 50 \text{ m/s}$$

$$C_{p,min} = -6.29 = 1 - \left(\frac{V}{V_{\infty}}\right)^2$$

$$V = 135 \text{ m/s}$$

Q.No. 35 A symmetrical airfoil section produces a lift coefficient of 0.53 at an angle of attack of 5 degrees measured from its chord line. An untwisted wing of elliptical planform and aspect ratio 6 is made of this airfoil. At an angle of attack of 5 degrees relative to its chordal plane, this wing would produce a lift coefficient of

- (A) 0.53 (B) 0.48 (C) 0.40 (D) 0.36

Sol. C

$$C_L = 0.53$$

For symmetrical airfoil, $C_{L0} = 0$

$$a_0 = \frac{\partial C_L}{\partial \alpha} = \frac{0.53-0}{5-0} = 0.106 \text{ per degree}$$

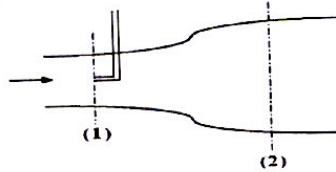
$$e=1, AR = 6$$

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$$\left(\frac{\partial C_L}{\partial \alpha}\right)_{wing} = a = \frac{a_0}{1 + \frac{a_0}{\pi e AR}} = 0.080 \text{ per degree}$$

$$C_L \text{ (For wing)} = a \cdot (\alpha - \alpha_{L=0}) = 0.08 \times 5 = 0.40$$

Q.No. 36 Consider an ideal flow of density ρ through a variable area duct as shown in the figure below :



Let the cross-sectional areas at sections (1) and (2) be A_1 and A_2 respectively. The velocity measured at section (1) using a Pitot static probe is V_1 . Then the static pressure drop $p_2 - p_1$ is

(A) $-\frac{1}{2}\rho\left(1 - \frac{A_1^2}{A_2^2}\right)V_1^2$

(B) $\frac{1}{2}\rho\left(1 - \frac{A_1^2}{A_2^2}\right)V_1^2$

(C) $\frac{1}{2}\rho\left(1 + \frac{A_1^2}{A_2^2}\right)V_1^2$

(D) $-\frac{1}{2}\rho\left(1 + \frac{A_1^2}{A_2^2}\right)V_1^2$

Sol. B

Continuity equation,

$$A_1 V_1 = A_2 V_2 \dots (1)$$

Bernoulli equation,

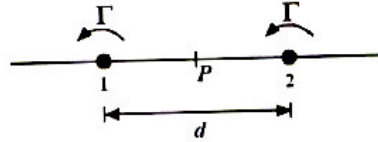
$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \dots (2)$$

So from equations (1) and (2),

$$P_2 - P_1 = \frac{1}{2}\rho V_1^2 \left[1 - \frac{A_1^2}{A_2^2}\right]$$

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Q.No. 37 Two vortices of the same strength and sign are placed a distance d apart as shown below. Assume that the vortices are free to move and the fluid is ideal.



Which of the following statements is true ?

- (A) Vortices 1 and 2 spiral inwards with an initial angular speed $\frac{\Gamma}{2\pi d^2}$ to finally merge and form one vortex of twice the strength.
- (B) Vortices 1 and 2 spiral inwards with an initial angular speed $\frac{\Gamma}{\pi d^2}$ to finally merge and form one vortex of twice the strength.
- (C) Vortices 1 and 2 perpetually revolve about the midpoint P with radius of revolution $\frac{d}{2}$ and angular speed $\frac{\Gamma}{2\pi d^2}$.
- (D) Vortices 1 and 2 perpetually revolve about the midpoint P with radius of revolution $\frac{d}{2}$ and angular speed $\frac{\Gamma}{\pi d^2}$.

Sol. D

Point 1 induces velocity at point 2 with magnitude of $\frac{\Gamma}{2\pi d}$ upwards.

Point 2 induces velocity at point 1 with magnitude of $\frac{\Gamma}{2\pi d}$ downwards.

The motion of these two velocities is circulatory about point p .

$$\text{Angular velocity, } \omega = \frac{v}{R} = \frac{\Gamma}{\pi d^2}$$

Q.No. 38 The laminar boundary layer over a large flat plate held parallel to the flow is 7.2 mm thick at a point 0.33 m downstream of the leading edge. If the free stream speed is increased by 50%, then the new boundary layer thickness at this location will be approximately

- (A) 10.8 mm (B) 8.8 mm (C) 5.9 mm (D) 4.8 mm

Sol. C

At 0.33 m, $\delta_1 = 7.2$ mm

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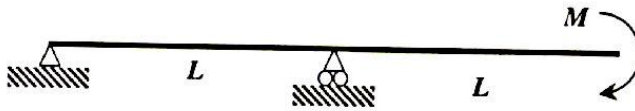
Boundary layer, $\delta = \frac{5x}{\sqrt{Re_x}}$; $Re_x = \frac{\rho V x}{\mu}$

So, $\delta \propto \frac{1}{\sqrt{V}}$

$\delta_1 = \frac{k}{\sqrt{V_1}}$

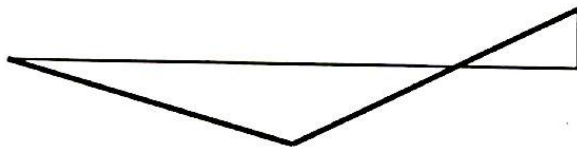
Now, $\delta_2 = \frac{k}{\sqrt{V_2}} = \frac{k}{\sqrt{V_1 + \frac{V_1}{2}}} = \frac{\delta_1}{\sqrt{3/2}} = 5.88 \text{ mm}$

Q.No. 39 Consider a simply supported beam of length $2L$ with an overhang of length L , loaded by an end moment M , as shown below.



The bending moment distribution for this beam is

(A)



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(B)



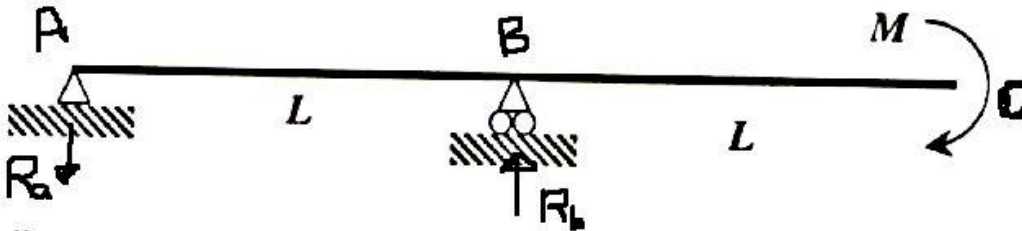
(C)



(D)



Sol. B



$$R_a + R_b = 0 \dots (1)$$

$$\sum M_a = 0 = R_b \times L - M = 0 \dots (2)$$

$$R_b = \frac{M}{L}$$

$$R_a = -\frac{M}{L} = \frac{M}{L} \downarrow$$

From RHS,

CB section –

$$M_x = -M \text{ (hogging)}$$

$$M_c = -M$$

$$M_b = -M$$

BM Section –

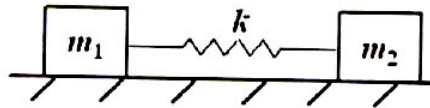
$$M_x = -M + R_b(x - L)$$

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$$M_b = -M$$

$$M_a = 0$$

Q.No. 40 For the spring-mass system shown below, the natural frequencies are



(A) 0 and $\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$

(B) 0 and $\sqrt{\frac{k(m_1 + m_2)}{2m_1 m_2}}$

(C) 0 and $\sqrt{\frac{k}{(m_1 + m_2)}}$

(D) 0 and $\sqrt{\frac{k}{2(m_1 + m_2)}}$

Sol. A

For 2 degree of spring mass system,

$$m_1 m_2 \omega^4 - [m_1(k_3 + k_2) + m_2(k_1 + k_2)] \omega^2 + [k_1 k_2 + k_2 k_3 + k_3 k_1] = 0$$

Here, $k_1 = 0$, $k_2 = k$, $k_3 = 0$

$$m_1 m_2 \omega^4 - [m_1 k + m_2 k] \omega^2 = 0$$

$$\omega = 0 \text{ and } \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

Q.No. 41 The buckling load for a simply supported column of rectangular cross section of dimensions 1 cm × 1.5 cm and length 0.5 m made of steel ($E = 210 \times 10^9 \text{ N/m}^2$) is approximately

(A) 10 kN

(B) 4 kN

(C) 23 kN

(D) 46 kN

Sol. A

$$L = 0.5 \text{ m, } E = 210 \times 10^9 \text{ N/m}^2$$

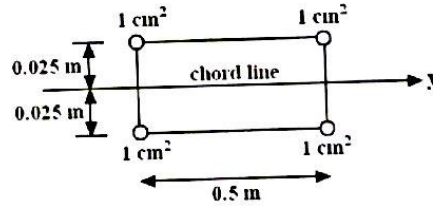
Rectangular c/s dimensions ($b \times d$) = 1 cm × 1.5 cm

$$I = \frac{db^3}{12} = \frac{(1.5 \times 10^{-2}) \times (1 \times 10^{-2})^3}{12} = 0.125 \times 10^{-8} \text{ m}^4$$

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For a simply supported column,

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 10.36 \text{ kN}$$

Q.No. 42 A wing root cross section is idealized using lumped areas (booms) as shown below.



The wing root bending moment in steady level flight is $M_y = 10 \text{ N-m}$. If the airplane flies at a load factor $n = 3.5$, the maximum bending stress at the root is

(A) $1 \times 10^6 \text{ N/m}^2$
(C) $7 \times 10^6 \text{ N/m}^2$

(B) $3.5 \times 10^6 \text{ N/m}^2$
(D) $0.286 \times 10^6 \text{ N/m}^2$

Sol. B

$$M_y = 10 \text{ N-m}$$

$$n = 3.5$$

$$I_{yy} = \int dA \cdot x^2 = 1 \times 10^{-4} \times (0.025)^2 \times 4 = 2.5 \times 10^{-7} \text{ m}^4$$

$$\sigma_{zz} = \frac{M_y}{I_{yy}} \cdot x = 10^6 \text{ N/m}^2$$

$$n = \frac{L}{W} = \frac{\text{upward net force}}{\text{weight}}$$

$$\therefore n = 1$$

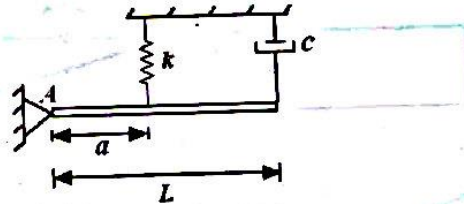
$$L = W$$

$$\text{So } \sigma_{max} \text{ (at } n = 1) = 10^6 \text{ N/m}^2$$

$$\text{At } n = 3.5, \sigma_{max} = 3.5 \times 10^6 \text{ N/m}^2$$

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Q.No. 43 A uniform rigid bar of mass $m = 1$ kg and length $L = 1$ m is pivoted at A. It is supported by a spring of stiffness $k = 1$ N/m and a viscous damper of damping constant $C = 1$ N-s/m, with $a = \frac{1}{\sqrt{3}}$ m as shown below. The moment of inertia of the rigid bar is $I_A = \frac{mL^2}{3}$.



The system is

- (A) overdamped
- (B) underdamped with natural frequency $\omega_n = 1$ rad/s
- (C) critically damped
- (D) underdamped with natural frequency $\omega_n = 2$ rad/s

Sol. B

$$m = 1 \text{ kg}, L = 1 \text{ m}, k = 1 \text{ N/m}, C = 1 \text{ N-s/m}, a = \frac{1}{\sqrt{3}} \text{ m}$$

$$I_A = \frac{mL^2}{3}$$

$$I_A \ddot{\theta} + (ka\theta).a = 0$$

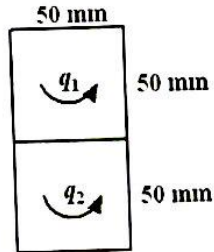
$$\omega_n = \sqrt{\frac{ka^2}{I_A}} = 1 \text{ rad/s}$$

$$C_c = 2m\omega_n = 2 \text{ N-s/m}$$

$$\text{Damping factor, } \xi = \frac{C}{C_c} = \frac{1}{2} < 1 \rightarrow \text{under damped}$$

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Q.No. 44 A 2-celled tube with wall thickness 0.5 mm is subjected to a torque of 10 N-m. The resulting shear flows in the two cells are q_1 and q_2 as shown below.



The torque balance equation (Bredt-Batho formula) for this section leads to

(A) $q_1 - q_2 = 2000 \text{ N/m}$

(B) $q_1 + 2q_2 = 2000 \text{ N/m}$

(C) $q_1 + q_2 = 2000 \text{ N/m}$

(D) $2q_1 + q_2 = 2000 \text{ N/m}$

10/16

Sol. C

$$T = 10 \text{ N-m}, t = 0.5 \text{ mm}$$

$$T = 2A_1q_1 + 2A_2q_2$$

$$A_1 = A_2 = 50 \times 50 \times 10^{-6} \text{ m}^2$$

$$q_1 + q_2 = 2000 \text{ N/m}$$

Q.No. 45 The value of the integral $\int_0^{\pi} \frac{dx}{1+x+\sin x}$ evaluated using the trapezoidal rule with two equal intervals is approximately

(A) 1.27

(B) 1.81

(C) 1.41

(D) 0.71

Sol. C

$$I = \int_0^{\pi} \frac{dx}{1+x+\sin x}$$

By trapezoidal rule,

$$I_{\text{trap.}} = \int_{x_0}^{x_0+nh} f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$f(x) = \frac{1}{1+x+\sin x}$$

$$h = \frac{\pi-0}{2} = \frac{\pi}{2}$$

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x	0	$\frac{\pi}{2}$	π
y	1	0.28004	0.24145

$$I_{trap.} = 1.41$$

Q.No. 46 The product of the eigenvalues of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$ is

(A) 20

(B) 24

(C) 9

(D) 17

Sol. D

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

Eigen values $\rightarrow \lambda_1, \lambda_2, \lambda_3$

$$\lambda_1 \times \lambda_2 \times \lambda_3 = |A| = 24 + 1 + 1 - 3 - 4 - 2 = 17$$

Q.No. 47 In the interval $1 \leq x \leq 2$, the function $f(x) = e^{\pi x} + \sin \pi x$ is

(A) maximum at $x = 1$ (B) maximum at $x = 2$ (C) maximum at $x = 1.5$

(D) monotonically decreasing

Sol. B

$$f(x) = e^{\pi x} + \sin \pi x$$

$$f'(x) = 0, \text{ for max. and min.}$$

Now just check manually for fast solving question,

$$f(1) = e^\pi$$

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$$f(2) = e^{2\pi}$$

$$f(1.5) = e^{\frac{3}{2}\pi} - 1$$

So it is max. at $x = 2$ in $[1, 2]$ interval.

Q.No. 48 The inverse Laplace transform of $F(s) = \frac{(s+1)}{(s+4)(s-3)}$ is

(A) $\frac{3}{7}e^{4t} + \frac{4}{7}e^{-3t}$

(B) $\frac{3}{7}e^{-4t} + \frac{4}{7}e^{3t}$

(C) $\frac{5}{7}e^{-4t} + \frac{6}{7}e^{3t}$

(D) $\frac{5}{7}e^{4t} + \frac{6}{7}e^{-3t}$

Sol. B

$$F(s) = \frac{s+1}{(s+4)(s-3)}$$

$$\frac{s+1}{(s+4)(s-3)} = \frac{A}{s+4} + \frac{B}{s-3}$$

$$A = 3/7$$

$$B = 4/7$$

$$F(s) = \frac{s+1}{(s+4)(s-3)} = \frac{3/7}{s+4} + \frac{4/7}{s-3}$$

$$L^{-1}\{F(s)\} = \frac{3}{7}L^{-1}\left\{\frac{1}{s+4}\right\} + \frac{4}{7}L^{-1}\left\{\frac{1}{s-3}\right\}$$

$$= \frac{3}{7}e^{-4t} + \frac{4}{7}e^{3t}$$

Q.No. 49 The linear system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ has

(A) no solution

(B) infinitely many solutions

(C) a unique solution $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(D) a unique solution $\mathbf{x} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

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Sol. A

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \text{ and } b = \begin{Bmatrix} 3 \\ 3 \end{Bmatrix}$$

$$A:b = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -3 \end{bmatrix} \rightarrow (R_2 \rightarrow R_2 - 2R_1)$$

$$\text{So, } \rho(A) = 1$$

$$\rho(A:b) = 2$$

So, No solution

Q.No.50 The correct iterative scheme for finding the square root of a positive real number R using the Newton Raphson method is

$$(A) x_{n+1} = \sqrt{R}$$

$$(B) x_{n+1} = \frac{1}{2} \left(x_n + \frac{R}{x_n} \right)$$

$$(C) x_{n+1} = \frac{1}{2} (\sqrt{x_n} + \sqrt{x_{n-1}})$$

$$(D) x_{n+1} = \frac{1}{2} (\sqrt{R} + x_n)$$

Sol. B

$$x^2 = R$$

$$f(x) = x^2 - R$$

$$f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \quad (\text{Newton - Raphson method})$$

$$x_{n+1} = x_n - \frac{x_n^2 - R}{2x_n} = \frac{1}{2} \left[x_n + \frac{R}{x_n} \right]$$

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Common Data Questions

Common Data for Questions 51 and 52:

The roots of the characteristic equation for the longitudinal dynamics of a certain aircraft are: $\lambda_1 = -0.02 + 0.2i$; $\lambda_2 = -0.02 - 0.2i$; $\lambda_3 = -2.5 + 2.6i$; $\lambda_4 = -2.5 - 2.6i$, where $i = \sqrt{-1}$.

Q.No. 51 The pair of eigenvalues that represent the phugoid mode is

- (A) λ_1 and λ_3 (B) λ_2 and λ_4 (C) λ_3 and λ_4 (D) λ_1 and λ_2

Sol. D

For phugoid mode, Eigen values are smaller than short period mode that is λ_1 and λ_2 .

Q.No. 52 The short period damped frequency is

- (A) 2.6 rad/s (B) 0.2 rad/s (C) 2.5 rad/s (D) 0.02 rad/s

Sol. A

For short period, the Eigen values have largest magnitude and imaginary parts.

$$\rightarrow S^2 - (\lambda_3 + \lambda_4).S + \lambda_3 \times \lambda_4 = 0$$

$$S^2 + 5S + 13.01 = 0$$

$$\omega_{n_s}^2 = 13.01$$

$$\omega_{n_s} = 3.606 \text{ rad/s}$$

$$2\xi\omega_{n_s} = 5$$

$$\xi = 0.693$$

$$\omega_{d_s} = \omega_{n_s} \times \sqrt{1 - \xi^2} = 2.59 \text{ rad/s}$$

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Common Data for Questions 53 and 54:

Consider the vector field $\vec{A} = (y^3 + z^3)\hat{i} + (x^3 + z^3)\hat{j} + (x^3 + y^3)\hat{k}$ defined over the unit sphere $x^2 + y^2 + z^2 = 1$.

Q.No. 53 The surface integral (taken over the unit sphere) of the component of \vec{A} normal to the surface is

- (A) π (B) 1 (C) 0 (D) 4π

Sol. C

$$\vec{A} = (y^3 + z^3)\hat{i} + (x^3 + z^3)\hat{j} + (x^3 + y^3)\hat{k}$$

By Gauss Divergence theorem,

$$\oiint (\vec{A} \cdot \vec{n}) ds = \iiint (\nabla \cdot \vec{A}) dV$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(y^3 + z^3) + \frac{\partial}{\partial y}(x^3 + z^3) + \frac{\partial}{\partial z}(x^3 + y^3) = 0$$

So, $\oiint (\vec{A} \cdot \vec{n}) ds = 0$

Q.No. 54 The magnitude of the component of \vec{A} normal to the spherical surface at the point $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ is

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{3}$ (D) $\frac{4}{3}$

Sol. B

$$\phi = x^2 + y^2 + z^2 - 1$$

Normal vector to the surface,

$$\begin{aligned} \nabla \phi &= \frac{\partial}{\partial x}(x^2 + y^2 + z^2 - 1) \hat{i} + \frac{\partial}{\partial y}(x^2 + y^2 + z^2 - 1) \hat{j} + \frac{\partial}{\partial z}(x^2 + y^2 + z^2 - 1) \hat{k} \\ &= 2x\hat{i} + 2y\hat{j} + 2z\hat{k} \end{aligned}$$

At $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ -

$$\text{Unit vector, } \vec{n} = \frac{\nabla \phi}{|\nabla \phi|} = \frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{\sqrt{4x^2 + 4y^2 + 4z^2}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

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At $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ -

$$\vec{A} \cdot \vec{n} = \frac{(y^3+z^3)}{\sqrt{3}} + \frac{(x^3+z^3)}{\sqrt{3}} + \frac{(x^3+y^3)}{\sqrt{3}} = \frac{2}{3}$$

Common Data for Questions 55 and 56:

The partial differential equation for the torsional vibration of a shaft of length L , torsional rigidity GJ , and mass polar moment of inertia per unit length I , is $I \frac{\partial^2 \theta}{\partial t^2} = GJ \frac{\partial^2 \theta}{\partial x^2}$, where θ is the twist.

Q.No. 55 If the shaft is fixed at both ends, the boundary conditions are:

- (A) $\frac{\partial \theta}{\partial x} \Big|_{x=0} = 0$ and $\frac{\partial \theta}{\partial x} \Big|_{x=L} = 0$ (B) $\theta(0) = 0$ and $\theta(L) = 0$
 (C) $\frac{\partial \theta}{\partial x} \Big|_{x=0} = 0$ and $\theta(L) = 0$ (D) $\theta(0) = 0$ and $\frac{\partial \theta}{\partial x} \Big|_{x=L} = 0$

Sol. B

Note – Go through vibration of shaft.

AE

Q.No. 56 If the n^{th} mode shape of torsional vibration of the above shaft is $\sin\left(\frac{n\pi x}{L}\right)$ then the n^{th} natural frequency of vibration, i.e., ω_n , is given by

- (A) $\omega_n = \frac{n\pi}{L} \sqrt{\frac{GJ}{I}}$ (B) $\omega_n = \frac{(2n+1)\pi}{2L} \sqrt{\frac{GJ}{I}}$
 (C) $\omega_n = \frac{n\pi}{2L} \sqrt{\frac{GJ}{I}}$ (D) $\omega_n = \frac{(2n+1)\pi}{L} \sqrt{\frac{GJ}{I}}$

Sol. B

$$\cos\left(\frac{\omega}{c} L\right) = \cos(2n+1) \frac{\pi}{2}$$

$$\omega = \frac{(2n+1)}{2L} \cdot c = \frac{(2n+1)\pi}{2L} \cdot \sqrt{\frac{GJ}{I}}$$

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Linked Answer Questions

Statement for Linked Answer Questions 57 and 58:

Air enters the combustor of a gas-turbine engine at a total temperature T_0 of 500 K. The air stream is split into two parts: primary and secondary streams. The primary stream reacts with fuel supplied at a fuel-air ratio of 0.05. The resulting combustion products are then mixed with the secondary air stream to obtain gas with total temperature of 1550 K at the turbine inlet. The fuel has a heating value of 42 MJ/kg. The specific heats of air and combustion products are taken as $c_p = 1$ kJ/kg/K.

Q.No. 57 If the sensible enthalpy of fuel is neglected, the temperature of combustion products from the reaction of primary air stream with fuel is approximately
(A) 2100 K (B) 3200 K (C) 2600 K (D) 1800 K

Sol. C

Air temperature at enter section of combustor, $T_{01} = 500$ K

At exit, $T_{03} = 1550$ K

$$f = \frac{\dot{m}_f}{\dot{m}_h} = 0.05 ; \dot{m}_a = \dot{m}_c + \dot{m}_h$$

$$\Delta H_{rp} = 42 \text{ MJ/kg}, c_p = 1 \text{ kJ/kg.K}$$

Energy equation in combustor (primary section),

$$\dot{m}_h \cdot h_{01} + \dot{m}_f \cdot \eta_b \cdot \Delta H_{rp} = (\dot{m}_h + \dot{m}_f) h_{02}$$

$$c_p \cdot T_{01} + f \cdot \Delta H_{rp} = (1+f) \cdot c_p \cdot T_{02}$$

$$T_{02} = 2476.91 \text{ K}$$

Q.No. 58 The approximate ratio of mass flow rates of the primary air stream to the secondary air stream required to achieve the turbine inlet total temperature of 1550 K is

- (A) 2:1 (B) 1:2 (C) 1:1.5 (D) 1:1

Sol. D

Energy equation in combustor (primary section + secondary section),

$$\Delta H_{01} + \Delta H_{02} = \Delta H_{03} \dots (1)$$

$$\dot{m}_c \cdot c_{p1} \cdot T_{01} + (\dot{m}_h + \dot{m}_f) c_{p2} \cdot T_{02} = (\dot{m}_a + \dot{m}_f) c_{p3} \cdot T_{03}$$

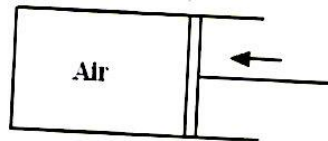
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$$c_{p1} = c_{p2} = c_{p3} = c_p$$

$$\frac{\dot{m}_h}{\dot{m}_c} = 1:1$$

Statement for Linked Answer Questions 59 and 60:

A piston compresses 1 kg of air inside a cylinder as shown



The rate at which the piston does work on the air is 3000 W. At the same time, heat is being lost through the walls of the cylinder at a rate of 847.5 W.

- Q.No. 59 After 10 seconds, the change in specific internal energy of the air is
 (A) 21,525 J/kg (B) -21,525 J/kg (C) 30,000 J/kg (D) -8,475 J/kg

Sol. A

$$\Delta Q = \Delta W + \Delta U$$

ΔW = work done by system

$$m = 1 \text{ kg}, \Delta \dot{W} = -3000 \text{ W}, \Delta \dot{Q} = -847.5 \text{ W}$$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta W}{\Delta t} + \frac{\Delta U}{\Delta t}$$

$$\frac{\Delta U}{\Delta t} = 2152.5$$

$$\Delta U = 21525 \text{ J}$$

For per kg mass, $\Delta U = 21525 \text{ J/kg}$

- Q.No. 60 Given that the specific heats of air at constant pressure and volume are $c_p = 1004.5 \text{ J/kg-K}$ and $c_v = 717.5 \text{ J/kg-K}$ respectively, the corresponding change in the temperature of the air is

- (A) 21.4 K (B) -21.4 K (C) 30 K (D) -30 K

Sol. C

$$C_p = 1004.5 \text{ J/kg-K}, C_v = 717.5 \text{ J/kg-K}$$

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$$\Delta U = mC_v\Delta T$$

$$\frac{\Delta U}{m} = C_v\Delta T$$

$$\Delta T = 30 \text{ K}$$

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